Arrow's Impossibility Theorem

Lecture 11
Section 1.6

Robb T. Koether

Hampden-Sydney College

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Outline

The Majority Criterion

Definition (Majority Criterion)

The Majority Criterion says that **if** a candidate has a majority of first-place votes, then that candidate should be the winner. (There may or may not be a majority candidate.)

- The Borda count method and Coombs' method may violate the Majority Criterion.

The Majority Criterion

Example (The Majority Criterion – Borda Count Method)

No. of Votes	8	6	5	3
1st	A	A	B	B
2nd	B	B	C	C
3rd	C	D	D	A
4th	D	C	A	D

- Does any candidate have a majority?
- Who wins by the Borda Count Method?

The Majority Criterion

Example (The Majority Criterion – Coomb's Method)

No. of Votes	8	6	5	2
1st	D	A	A	B
2nd	B	B	C	C
3rd	C	D	D	A
4th	A	C	B	D

- Does any candidate have a majority?
- Who wins by Coomb's Method?

The Majority Criterion

- The Borda count method and Coombs' method may violate the Majority Criterion.
- The other methods (I'm pretty sure) do not violate it.

Outline

The Condorcet Criterion

Definition (Condorcet Winner)

The Condorcet winner is a candidate who beat *every other candidate* in pairwise comparisons. (There may or may not be a Condorcet winner.)

Definition (Condorcet Criterion)

The Condorcet Criterion says that **if** there is a Condorcet winner, then that candidate should be the winner (by whatever method used).

The Condorcet Criterion

Example (The Condorcet Criterion – Borda Count Method)

No. of Votes	8	6	5	3
1st	A	A	B	B
2nd	B	B	C	C
3rd	C	D	D	A
3rd	D	C	A	D

- Is there a Condorcet winner?
- Who wins by the Borda Count Method?

The Condorcet Criterion

- The plurality method, Borda count method, plurality-with-elimination method, and Coombs' method may violate the Condorcet Criterion.
- The other method (I'm pretty sure) does not violate it.

Outline

The Monotonicity Criterion

Definition (Monotonicity Criterion)

The Monotonicity Criterion says that **if** candidate X is the winner, then X would still be the winner if a voter had placed X higher in his ranking.

The Monotonicity Criterion

Example (The Monotonicity Criterion – Plurality-with-Elimination Method)

	10	8	7	4
1st	B	A	C	C
2nd	C	B	A	B
3rd	A	C	B	A

- Who is the winner by the Plurality-with-Elimination Method?
- Suppose that the last four voters decided to rank B over C. Now who is the winner?

The Monotonicity Criterion

- The plurality-with-elimination Method may violate the Monotonicity Criterion.
- The other methods (I'm pretty sure) do not violate it.

Outline

The Independence-of-Irrelevant-Alternatives Criterion

Definition (Independence-of-Irrelevant-Alternatives Criterion)

The independence-of-irrelevant-alternatives criterion (IIA) says that **if** candidate X is the winner, then X would still be the winner if one or more of the losing candidates had not been in the race.

The Independence-of-Irrelevant-Alternatives Criterion

Example (The Independence-of-Irrelevant-Alternatives Criterion – Plurality-with-Elimination Method)

	10	8	7	4
1st	B	A	C	C
2nd	C	B	A	B
3rd	A	C	B	A

- Who is the winner by the Plurality-with-Elimination Method?
- Suppose that candidate C drops out. Now who is the winner?

The Independence-of-Irrelevant-Alternatives Criterion

- The plurality method and the plurality-with-elimination method may violate the IIA Criterion.
- The other methods (I'm pretty sure) do not violate it.

Outline

Arrow's Impossibility Theorem

Theorem (Arrow's Impossibility Theorem)

If there are at least 3 candidates, then there is no voting method that cannot violate any of the four desired properties (Majority, Condorcet, Monotonicity, Independence of Irrelevant Alternatives).

Outline

Assignment

Assignment

- Chapter 1 Exercises 51, 52, 53, 54, 55, 56.